Comparison of Hybrid Methods for Global Variance Reduction in Shielding Calculations

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Monte Carlo Transport – Variance Reduction
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Introduction

• Hybrid Methods
  – Use discrete ordinates estimates to construct variance reduction parameters for a detailed Monte Carlo calculation
  – $S_N$ estimates: forward, adjoint or both

• Variance reduction parameters
  – Importance map for weight windows, biased source
  – Optimize the figure-of-merit (FOM) of one or more tallies
    • Single detector response
    • Flux/response over a portion of the problem
    • Flux/response over the entire problem space
Monte Carlo for Global Solutions

- **Mesh tallies**
  - Consist of $I \times J \times K$ voxels
  - Range of flux values
  - Range of uncertainties

Neutron dose from a criticality accident

Implicit capture only 18 hours
Monte Carlo for Global Solutions

• Mesh tallies
  - Consist of \( I \times J \times K \) voxels
  - Range of flux values
  - Range of uncertainties

Neutron dose from a criticality accident

Hybrid
18 hours
Methods for Global Variance Reduction

• Cooper’s Method

• GFWW/GRWW

• Van Wijk’s Methods

• Becker’s Methods

• FW-CADIS
Cooper’s Method

- **Goal:** uniform relative uncertainties
  - Make the Monte Carlo particle density $m(\vec{r})$ constant
  - Physical density $n(\vec{r})$ is related to the average weight $\bar{w}(\vec{r})$ by
    \[ n(\vec{r}) = \bar{w}(\vec{r}) \cdot m(\vec{r}) \]

- **So, use forward deterministic estimate of flux $\phi(\vec{r})$**
  - and make $\bar{w}(\vec{r}) \propto \phi(\vec{r})$
    \[ \bar{w}(\vec{r}) = \frac{\phi(\vec{r})}{\max(\phi(\vec{r}))} \]
  - Space-only method
Global Flux/Response Weight Windows

- Extension of Cooper’s method to space/energy
- Use forward discrete ordinates estimate of $\phi(\vec{r}, E)$
- To optimize MC $\phi(\vec{r}, E)$ everywhere

$$\overline{w}(\vec{r}, E) \propto \phi(\vec{r}, E)$$

- To optimize MC $D(\vec{r}) = \int_E f(E) \phi(\vec{r}, E) dE$ everywhere

$$\overline{w}(\vec{r}, E) \propto \frac{\int_E f(E) \phi(\vec{r}, E) dE}{f(E)} = \frac{D(\vec{r})}{f(E)}$$
Van Wijk’s Methods

- Developed an iterative scheme with MCNP
- Estimates could be found using deterministic solution

- Flux-based method: \[ \bar{w}(\vec{r}) = \frac{\phi(\vec{r})}{\max(\phi(\vec{r}))} \]

- Relative error is \( Re(\vec{r}) \propto 1/\sqrt{\phi(\vec{r})} \)

- \( Re \)-based method:
  \[ \bar{w}(\vec{r}) = \frac{\min(Re(\vec{r}))}{Re(\vec{r})} \]
Becker’s Methods

- Uses forward and adjoint estimates
- Different for detector, region, or global problems
- Optimize either $\phi(\vec{r}, E)$ or response function $f(E)$

Compute the forward flux estimate
$$\phi(\vec{r}, E)$$

Construct the adjoint source -
for optimizing flux use
$$q^+(\vec{r}, E) = \frac{1}{\phi(\vec{r}, E)}$$

or

for optimizing response use
$$q^+(\vec{r}, E) = \frac{f(E)}{\int_0^\infty f(E) \phi(\vec{r}, E) dE}$$

Compute the adjoint flux estimate
$$\phi^+(\vec{r}, E)$$

Construct the contribution flux
$$\phi^c(\vec{r}, E) = \phi(\vec{r}, E) \phi^+(\vec{r}, E)$$

Find the space-only contribution flux
$$\phi^c(\vec{r}) = \int_0^\infty \phi^c(\vec{r}, E) dE$$

Construct the weight windows
$$\bar{w}(\vec{r}, E) = \frac{\phi^c(\vec{r})}{\phi^+(\vec{r}, E)}$$
FW-CADIS

• Uses forward and adjoint estimates
• Same for detector, region, or global problems
• Optimize either $\phi(\vec{r}, E)$ or response function $f(E)$

1. Compute the forward flux estimate
2. Construct the adjoint source -
   - for optimizing flux use
   - or
   - for optimizing response use
3. Compute the adjoint flux estimate
4. Compute the response estimate
5. Construct the weight windows
6. Construct the biased source

$\phi(\vec{r}, E) = \frac{\int_0^\infty f(E) \phi(\vec{r}, E) dE}{\int_0^\infty f(E) \phi(\vec{r}, E) dE}$
Mesh Tally Metrics

• Look at the relative error, $r_i$, in each of the $V$ voxels

• Form a distribution of relative uncertainties

• Mean relative uncertainty $\bar{r} = \frac{1}{V} \sum r_i$
  - Should be $\propto 1/\sqrt{N}$ if each voxel is $\propto 1/\sqrt{N}$

• Tests:
  - Is the fraction of voxels with score, $\zeta$, constant?
  - Is $\bar{r} \propto 1/\sqrt{N}$?
  - Is the variance of $\bar{r} \propto 1/N$?
  - Is the FOM, $1/(\bar{r}^2 T)$, constant?

Based on:
Simple Shielding Problem

- **420 × 220 × 240 cm lead rooms**
- **Source: 1 Ci, 14 MeV, point**
- **Optimize $D(\vec{r})$ everywhere**

<table>
<thead>
<tr>
<th>Method</th>
<th>Time (min) for adj</th>
<th>MC</th>
<th>Total</th>
</tr>
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<tbody>
<tr>
<td>Analog</td>
<td>420</td>
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<td>420</td>
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<tr>
<td>Cooper</td>
<td>8.9</td>
<td>50.9</td>
<td>60</td>
</tr>
<tr>
<td>van Wijk (Re)</td>
<td>8.9</td>
<td>50.1</td>
<td>59</td>
</tr>
<tr>
<td>GRWW</td>
<td>8.9</td>
<td>51.0</td>
<td>60</td>
</tr>
<tr>
<td>Becker</td>
<td>8.9</td>
<td>7.4</td>
<td>40.8</td>
</tr>
<tr>
<td>FW-CADIS</td>
<td>8.9</td>
<td>7.4</td>
<td>40.3</td>
</tr>
</tbody>
</table>

- **Mesh Tally:**
  - $84 \times 44 \times 48$ uniform
- **Deterministic**
  - grid: $51 \times 35 \times 37$ non-uniform
  - $S_8$, $P_3$
  - 27 groups
Simple Shielding Problem

analog

Van Wijk

GRWW

Becker

FW-CADIS
Simple Shielding Problem

analog

Van Wijk

GRWW

Becker

FW-CADIS
# Simple Shielding Problem

<table>
<thead>
<tr>
<th>Method</th>
<th>Time (min)</th>
<th>FOM</th>
<th>Stat. Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>for adj MC Total</td>
<td>ζ</td>
<td>R (/min)</td>
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<tr>
<td>Becker</td>
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<tr>
<td>FW-CADIS</td>
<td>8.9 7.4 40.3 57</td>
<td>1.0000</td>
<td>2.03E-02</td>
</tr>
</tbody>
</table>

**PDF of relative uncertainties**

**CDF of relative uncertainties**
Deep Penetration Shielding Problem

- 860 × 460 × 460 cm concrete structure
- Nuclear fuel: 80 × 80 × 80 cm cube
- Optimize $\phi(\vec{r}, E)$ everywhere

<table>
<thead>
<tr>
<th>Method</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>for</td>
</tr>
<tr>
<td>Analog</td>
<td></td>
</tr>
<tr>
<td>Cooper</td>
<td>29.2</td>
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<tr>
<td>van Wijk (Re)</td>
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<tr>
<td>GFWW</td>
<td>29.2</td>
</tr>
<tr>
<td>Becker</td>
<td>29.2</td>
</tr>
<tr>
<td>FW-CADIS</td>
<td>29.2</td>
</tr>
</tbody>
</table>

- **Mesh Tally:**
  - 86 × 46 × 46 uniform
  - 27 groups

- **Deterministic:**
  - grid: 106 × 58 × 50 non-uniform
  - $S_8$, $P_3$
  - 27 groups
Deep Penetration Shielding Problem

1 MeV
Deep Penetration Shielding Problem

1 MeV

- analog
- Van Wijk
- Becker
- Cooper
- GFWW
- FW-CADIS
### Deep Penetration Shielding Problem

<table>
<thead>
<tr>
<th>Method</th>
<th>Time (min)</th>
<th>FOM</th>
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<tr>
<td>Becker</td>
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<td>0.9997</td>
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<tr>
<td>FW-CADIS</td>
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<td>1.0000</td>
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<th>3</th>
<th>4</th>
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<tr>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Cooper</td>
<td>X</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>van Wijk (Re)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>GFWW</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
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<tr>
<td>Becker</td>
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<tr>
<td>FW-CADIS</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

### Statistical Tests

PDF of relative uncertainties

CDF of relative uncertainties
Deep Penetration Shielding Problem

- One location – look at energy

![Graph showing flux per unit lethargy and relative uncertainty in flux as functions of neutron energy (eV). The graph compares different methods: analog, Cooper, van WijkRe, GFWW, Becker, FW-CADIS.](image)
Becker’s Challenge

- 300 × 300 × 300 cm
- Source: fission neutrons
- Optimize $\phi(\vec{r})$ everywhere

<table>
<thead>
<tr>
<th>Method</th>
<th>Time (min) for</th>
<th>Adj</th>
<th>MC</th>
<th>Total</th>
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<tr>
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<td>1173</td>
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<td>van Wijk (Re)</td>
<td>38.6</td>
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<td>1202</td>
<td></td>
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<tr>
<td>GRWW</td>
<td>38.6</td>
<td>1184</td>
<td>1222</td>
<td></td>
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<tr>
<td>Becker</td>
<td>38.6 48.6</td>
<td>1118</td>
<td>1205</td>
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<td>FW-CADIS</td>
<td>38.6 48.7</td>
<td>1117</td>
<td>1204</td>
<td></td>
</tr>
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</table>

- Mesh Tally:
  - 30 × 30 × 30 uniform (1/8th)

- Deterministic
  - grid: 68 × 68 × 68 non-uniform
  - $S_8$, $P_3$
  - 27 groups
Becker’s Challenge Problem

analog

Van Wijk

Becker

Cooper

GRWW

FW-CADIS
Becker’s Challenge Problem

analog

Van Wijk

Becker

Cooper

GRWW

FW-CADIS
### Becker’s Challenge Problem

27,000 voxels

#### Time (min) for adj MC Total

<table>
<thead>
<tr>
<th>Method</th>
<th>Time (min)</th>
<th>adj MC</th>
<th>Total</th>
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<tbody>
<tr>
<td>Analog</td>
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<td>1203</td>
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<td>Cooper</td>
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<td>1212</td>
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<td>van Wijk (Re)</td>
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<td>GRWW</td>
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<tr>
<td>Becker</td>
<td>38.6</td>
<td>1118</td>
<td>1205</td>
</tr>
<tr>
<td>FW-CADIS</td>
<td>38.6</td>
<td>1117</td>
<td>1204</td>
</tr>
</tbody>
</table>

#### FOM Stat. Test

<table>
<thead>
<tr>
<th>Method</th>
<th>$\zeta$</th>
<th>$\bar{R}$ (min)</th>
<th>Stat. Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog</td>
<td>0.3428</td>
<td>2.64E-01</td>
<td>-</td>
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<tr>
<td>Cooper</td>
<td>1.0000</td>
<td>3.49E-01</td>
<td>X</td>
</tr>
<tr>
<td>van Wijk (Re)</td>
<td>0.9675</td>
<td>3.54E-01</td>
<td>X</td>
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<td>GRWW</td>
<td>1.0000</td>
<td>3.58E-01</td>
<td>X</td>
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<tr>
<td>Becker</td>
<td>1.0000</td>
<td>5.26E-02</td>
<td>X</td>
</tr>
<tr>
<td>FW-CADIS</td>
<td>1.0000</td>
<td>3.92E-02</td>
<td>X</td>
</tr>
</tbody>
</table>

#### PDF of relative uncertainties

![PDF of relative uncertainties](image1)

#### CDF of relative uncertainties

![CDF of relative uncertainties](image2)
MC Performance Benchmark

- 5 m diam reactor
- Source: from Keno
- Optimize $\phi(\vec{r})$

<table>
<thead>
<tr>
<th>Method</th>
<th>Time (min)</th>
<th>Total</th>
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</thead>
<tbody>
<tr>
<td>Analog</td>
<td>612</td>
<td>612</td>
</tr>
<tr>
<td>Cooper</td>
<td>16.9 585</td>
<td>602</td>
</tr>
<tr>
<td>van Wijk (Re)</td>
<td>16.9 586</td>
<td>602</td>
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<tr>
<td>GRWW</td>
<td>16.9 584</td>
<td>601</td>
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<tr>
<td>Becker</td>
<td>16.9 31.2</td>
<td>558</td>
</tr>
<tr>
<td>FW-CADIS</td>
<td>16.9 31.2</td>
<td>558</td>
</tr>
</tbody>
</table>

- **Mesh Tally:**
  - $27 \times 27 \times 27$ non-uniform
  - 27 groups

- **Deterministic:**
  - grid: $92 \times 92 \times 80$ non-uniform
  - $S_4$, $P_3$
  - 27 groups
MC Performance Benchmark

- Analog
- Cooper
- Van Wijk
- GRWW
- Becker
- FW-CADIS
MC Performance Benchmark

analog

Van Wijk

Cooper

GRWW

Becker

FW-CADIS

Neutron Flux
Total Flux
Relative Uncertainty

9.0E-02 - 1.0E-01
8.0E-02 - 8.0E-02
7.0E-02 - 7.0E-02
6.0E-02 - 6.0E-02
5.0E-02 - 5.0E-02
4.0E-02 - 4.0E-02
3.0E-02 - 3.0E-02
2.0E-02 - 2.0E-02
1.0E-02 - 1.0E-02
0.0E00 - 1.0E-02

Scale: 100.0 cm

Scale: 100.0 cm

Scale: 100.0 cm

Scale: 100.0 cm

Scale: 100.0 cm

Scale: 100.0 cm
## MC Performance Benchmark

<table>
<thead>
<tr>
<th>Method</th>
<th>Time (min)</th>
<th>FOM</th>
<th>Stat. Test</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Analog</strong></td>
<td>612</td>
<td>0.6203</td>
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<tr>
<td><strong>Cooper</strong></td>
<td>16.9</td>
<td>0.7531</td>
<td>- - - -</td>
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<tr>
<td><strong>van Wijk (Re)</strong></td>
<td>16.9</td>
<td>0.7524</td>
<td>X - - -</td>
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<tr>
<td><strong>GRWW</strong></td>
<td>16.9</td>
<td>0.7531</td>
<td>X - - -</td>
</tr>
<tr>
<td><strong>Becker</strong></td>
<td>16.9</td>
<td>0.7531</td>
<td>X X X X</td>
</tr>
<tr>
<td><strong>FW-CADIS</strong></td>
<td>16.9</td>
<td>0.7531</td>
<td>X X X X</td>
</tr>
</tbody>
</table>

**Performance Metrics**:

- **PDF of Relative Uncertainties**
- **CDF of Relative Uncertainties**

- **Analog**
- **Cooper**
- **van Wijk (Re)**
- **GRWW**
- **Becker**
- **FW-CADIS**
Summary

• Forward/adjoint methods far superior to forward-only
  – Adjoint contains importance information
  – More flexible for semi-global problems
  – Forward-based methods spread particles around evenly

• FW-CADIS
  – Higher FOM’s than Becker’s global method on all problems
  – More flexible (same method for semi-global problems)

• Note that most problems are not global
  – Everything everywhere is usually not important